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THE ICING OF AIRCRAFT

By M. Robitzsch

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 1028

THE ICING OF AIRCRAFT*

By M. Robitzsch

The subject of the present article has already been discussed repeatedly from purely practical points of view (reference 1). All the more, it seems necessary to analyze the problem theoretically, since a study of the processes accompanying the formation of ice on solid bodies of itself comprises all factors involved and makes it possible to weigh the effect of each against the total effect.

The problem is to be treated from the very beginning.

Assume two air masses A and B separated by a boundary area G. Air mass A has a temperature T_1 ; in it the partial pressure of water vapor e_1 prevails. For air mass B the corresponding values of these quantities which are to change abruptly on surface G, are T_2 and e_2 . The unit mass of air A has a total heat content W_1 , built up additively from two components:

The perceptible heat content W_1' which is proportional to the absolute temperature T_1 of A. The proportionality factor is the specific heat c_p of (moist) air; as the c_p value differs very little for moist or dry air, it is introduced at ($c_p = 241$ cal/kg). Then

$$W_1' = c_p T_1$$

The latent heat content W_1'' which is proportional to the specific moisture of air s_1 (or, expressed in g/kg)

$$s_1 = c \frac{e_1}{B}$$

where $c = 622$ indicates the specific weight of water vapor referred to air = 1000, B the barometric pressure -

*"Über die Vereisung von Luftfahrzeugen." Beiträge zur Physik der freien Atmosphäre, vol. 18, no. 4, 1932, pp. 235-241.

expressed in the same measure as e_1 . The proportionality factor is the heat of evaporation of the water, $L = 606 \text{ cal/g}$; hence

$$W_1'' = L c \frac{e_1}{B}$$

Then the total heat content per unit mass of moist air W_1 is

$$W_1 = W_1' + W_1'' = c_p T_1 + L c \frac{e_1}{B} = c_p \left[T_1 + \frac{Lc}{c_p} \frac{e_1}{B} \right]$$

The bracketed quantity defines the equivalent temperatures θ_1 of the analyzed air mass. The same derivations hold for the unit mass of air B ; it affords

$$W_2 = W_2' + W_2'' = c_p \left[T_2 + \frac{Lc}{c_p} \frac{e_2}{B} \right]$$

Next we assume that air mass m moves from B toward A through the unit surface E of boundary area G . The continuity stipulates that the same air mass m moves back from A toward B . Supposing this also takes place through the surface element E . This process changes the total heat content of unit mass A in a well-defined manner. With \bar{W}_1 denoting this quantity after the interchange, we get

$$\bar{W}_1 = W_1 + m c_p \left[T_2 + \frac{Lc}{c_p} \frac{e_2}{B} \right] - m c_p \left[T_1 + \frac{Lc}{c_p} \frac{e_1}{B} \right]$$

or

$$W_1 - \bar{W}_1 = m c_p \left[\left(T_1 + \frac{Lc}{c_p} \frac{e_1}{B} \right) - \left(T_2 + \frac{Lc}{c_p} \frac{e_2}{B} \right) \right]$$

Expressed in differential form - with respect to time - this formula can be written as

$$\frac{dW}{dt} = \frac{dm}{dt} c_p \left[\left(T_1 + \frac{Lc}{c_p} \frac{e_1}{B} \right) - \left(T_2 + \frac{Lc}{c_p} \frac{e_2}{B} \right) \right]$$

or, after regrouping of the terms, as

$$\frac{dW}{dt} = \frac{dm}{dt} c_p (T_1 - T_2) + \frac{dm}{dt} L \frac{c}{B} (e_1 - e_2)$$

The heat flow $\frac{dW}{dt}$ also consists accordingly of two components:

The first, $\frac{dP}{dt} = \frac{dm}{dt} c_p (T_1 - T_2)$ is sustained throughout the existence of a temperature difference $T_1 - T_2$ between A and B. It is unessential for moisture transport.

The second component $\frac{dQ}{dt} = \frac{dm}{dt} L \frac{c}{B} (e_1 - e_2)$ comprises the heat transport resulting from a flow of latent vapor heat. It is determinant for the interchange of moisture of the air masses A and B.

Next, assume that air mass A forms the "transitional layer" of a moist body the moist surface of which continuously gives off so much water vapor on the transitional layer that the partial pressure of water vapor in this transitional layer maintains the average value e_1 . Air mass B is to be so large that its specific partial pressure of water vapor e_2 is not changed by the inflow of the water vapor from A. Then

$$\frac{1}{L} \frac{dQ}{dt}$$

is the quantity of water $\frac{dV}{dt}$ passing through the transitional layer per unit surface per unit time, that is, the volume of water evaporating per unit surface of body per unit time. The formula reads

$$\frac{dV}{dt} = \frac{dm}{dt} \frac{c}{B} (e_1 - e_2)$$

If $e_2 > e_1$, condensation must occur on the body surface. The amount of water condensed on the unit surface of the body in unit time is then given by

$$\frac{dU}{dt} = \frac{dm}{dt} \frac{c}{B} (e_2 - e_1)$$

If water sublimates on the surface of the body, the evaporation heat L is replaced by the quantity $L + S$, S denoting the freezing heat of water (80 cal/g). So, by equal vapor pressure difference $e_2 - e_1$, the ratio of condensed to sublimated water is given by the fraction

$$\frac{L + S}{L} = \frac{686}{606} = 1.13$$

So far, no assumptions have been made concerning quantity e_1 and e_2 . Naturally, in the event of condensation, e_1 must be replaced by the saturation pressure E_1 of the water vapor at the temperature of the body surface. In the case of sublimation E_1 automatically changes to \bar{E}_1 , a quantity representing the saturation pressure of water vapor above ice.

In the present case, dealing with icing on aircraft; that is, solid bodies in the atmosphere, the sublimation formula

$$- \frac{1}{1.13} \frac{dV}{dt} = \frac{dU'}{dt} = \frac{dm}{dt} \frac{c}{B} (e_2 - \bar{E}_1)$$

must be interpreted.

The amount of ice sublimated per unit surface of the body in unit time is proportional to the "ventilation factor" $\frac{dm}{dt}$ (reference 2). By definition, this quantity

is the air mass dm passing into and out of the transitional layer in the time differential dt through the unit surface. This quantity, besides being a function of the speed of the ventilation flow, is at the same time substantially decided by the body form, the surface condition, and the direction of the stream. Its determination is more of an aerodynamic than a meteorological problem. On an airplane, for instance, the effect of the local variation of $\frac{dm}{dt}$ is such that the ice deposit is not of the same thickness everywhere; the deposition prefers the "exposed" parts of the structure.

The sublimating ice mass is, in addition, inversely proportional to the momentary barometric pressure. Hence

the sublimating ice mass is, other things being equal, greater in higher atmospheric layers than near the ground, or the same ice mass sublimates in higher atmospheric layers at a much lower vapor pressure difference than necessary in ground proximity.

The simplest method of estimating the vapor pressure difference $\Delta = e_2 - E_1$ is to take its amount from one of the conventional tables. But it can be much more clearly expressed if represented in relation to the following elements familiar to the meteorologist:

T_2 air temperature, degrees Centigrade

e_2 partial pressure of water vapor in air

E_2 saturation pressure of water vapor referred to water at temperature T_2

$R_2 = \frac{e_2}{E_2}$ saturation ratio in air

T_1 surface temperature of body in deg C; in the case in in point - sublimation processes - it is always negative

E_1 saturation pressure of water vapor referred to water at temperature T_1

\bar{E}_1 saturation pressure of water vapor referred to ice at temperature T_1

$R_1 = \frac{e_2}{\bar{E}_1}$ saturation ratio of air referred to ice at the temperature of the body

Then $e_2 = E_2 R_2 = \bar{E}_1 R_1$, by definition, and

$$\Delta = e_2 - \bar{E}_1 = \bar{E}_1 R_1 - \bar{E}_1 = \bar{E}_1 (R_1 - 1)$$

In this formula for Δ , \bar{E}_1 appears as pure function of the surface temperature T_1 of the body; while R_1 is a complex function of the above enumerated single elements. This quantity must be eliminated. For this purpose we put

$$R_1 = \frac{E_2 R_2}{\bar{E}_1}$$

which is a formula following from the definition of the saturation ratio; E_2 is expressed by

$$E_2 = E_1 + \frac{dE_1}{d\tau} (\tau_2 - \tau_1)$$

which affords a satisfactory approximation for small temperature intervals. From this, follows

$$R_1 = \frac{\left[E_1 + \frac{dE_1}{d\tau} (\tau_2 - \tau_1) \right] R_2}{\bar{E}}$$

or, after minor transformation:

$$R_1 - 1 = \frac{E_1 \left[1 + \frac{1}{E_1} \frac{dE_1}{d\tau} (\tau_2 - \tau_1) \right] R_2}{\bar{E}_1} - 1$$

Factor $\frac{1}{E_1} \frac{dE_1}{d\tau}$ is constant over a wide temperature range and has the numerical value 0.073, which, entered in the calculation, gives for the desired quantity Δ

$$\Delta = \bar{E}_1 (R_1 - 1) = E_1 [1 + 0.073 (\tau_2 - \tau_1)] R_2 - \bar{E}_1$$

E_1 and \bar{E}_1 are quantities the functional connection of which is given with τ_1 .

Hence Δ can be represented in a system of coordinates containing temperature τ_1 as abscissa, and the temperature difference between air and body surface $\tau_2 - \tau_1$ as ordinate, leaving the factor R_2 freely disposable.

The diagram shows two such representations side by side: The one on the left gives Δ for $R_2 = 1$, that on the right for $R_2 = 2$. The curves represent integral Δ values.

The left side of the diagram, which stipulates air saturated with water vapor, indicates that in this case the occurrence of sublimation processes of any considerable extent requires the air temperature to be

substantially higher than the temperature of the body surface. At lower body temperatures ice formation is possible because of the vapor pressure difference $E_1 - \bar{E}_1$, if the temperature of the body is above that of the air.

In air supersaturated with water vapor the conditions are markedly different - as shown in the right-hand side - for very high supersaturation in order to bring out the effect more plainly. In supersaturated air sublimation processes can therefore still occur if the body temperature is much higher than the air temperature (7° in the hypothetical case). If no temperature difference exists, sublimation processes already occur to an extent which in saturated air postulate temperature differences in excess of 6° between air and body. Without entering into a discussion as to whether such temperature differences occur, the forcible conclusion can be drawn that the formation of sublimation products is due less to saturated than to supersaturated air.

The study of temperature effect on the formation of sublimation products can be undertaken by means of a representation which reproduces the difference values of the Δ quantities of the right and left half of the diagram in a congenerous coordinate system. But even without such a representation, it is readily seen that the maximum differences of this nature occur at high body surface temperatures T_1 , that is, near the freezing point. Then it is seen that the formation of sublimation products occurs most abundantly in such air strata the temperature of which is close to the freezing point and supersaturated with water vapor.

The representation would be incomplete without the icing processes which are a result of the water in the atmosphere frequently in liquid state - subcooled at temperatures below the freezing point. Such conditions offer the greatest potentialities for icing. Aerologically, they are tied to air strata that form the exchange zones between damp cold air and equally damp warm air (wetting fog). Concerning the water content w which the unit air mass contains in liquid state in such cases, a number of measurements are available which afford some insight into the order of magnitude of this admixture. In any case, several grams per kilogram of air are involved, although for the time being, a theoretical study of such cases is impossible.

Even so, our considerations can be completed to the extent that the sublimation formula must be complemented by an additive term of the form

$$a \frac{dm}{dt} w$$

where a is a constant, $\frac{dm}{dt}$, as before, the air mass exchanged in unit time through the unit surface of the transitional layer; $\frac{dm}{dt} w$ is the amount of water in liquid state entering per unit time per unit surface into the transitional layer, an admixture which with the simultaneously outflowing air is not all removed again, because the partial amount a on the surface of the body freezes on striking it. Hence it can be seen that determinations of w have not merely a purely scientific, but also an eminently practical significance.

The arguments are equally applicable in modified form to the process of formation of precipitation elements of solid state of aggregation in the atmosphere. This question is to be discussed elsewhere.

Translation by J. Vanier,
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for Aeronautics.

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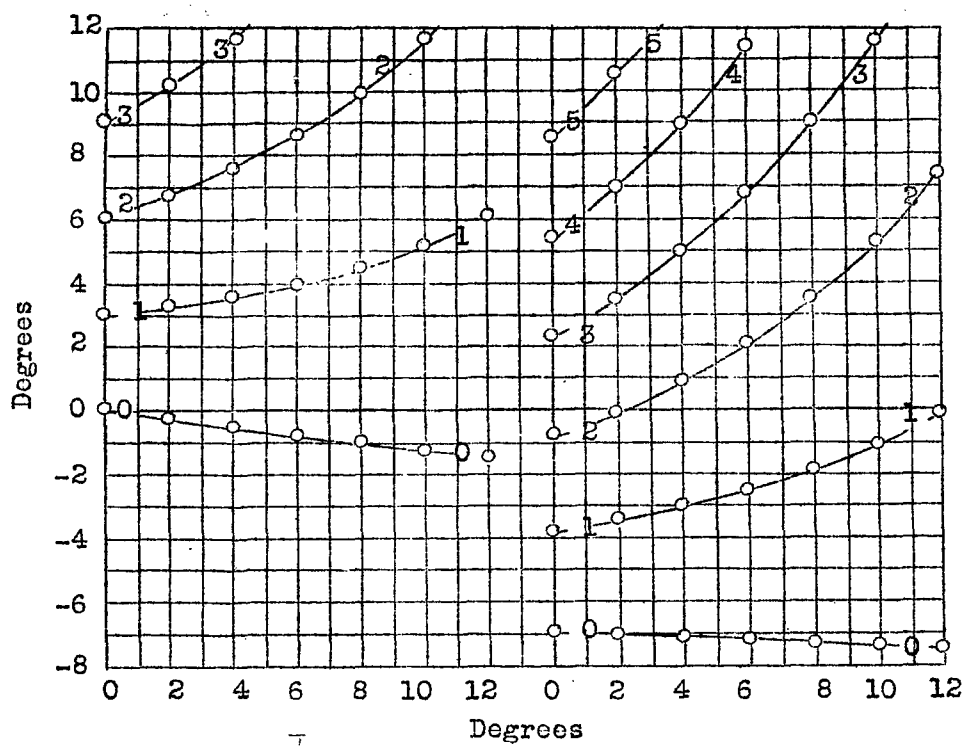


Figure 1

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